**Probability**

* **Probability experiment/trial** – a chance process that leads to will-defined result/outcomes
  + All possible outcomes are known before the experiment
  + **Sample space** – S = set of all possible (simple) outcomes of an experiment
    - A subset of S is an (random) event
  + E.g. experiment = flipping a coin
    - S = {H, T}
  + E.g. flipping 2 coins
    - S = {(HH), (HT), (TH), (TT)}
  + A tree diagram can be used to determine the sample space
  + Simple event – a single outcome
  + Compound event – more than one outcomes
  + Complement – all other outcomes in the sample space which are not in this event
* **Classical probability** – assumes all outcomes in the sample space are equally likely
  + P(E) = n(E)/n(S)
* **Empirical probability** – does not assume all outcomes in the sample space are equally likely
  + Relies on actual observation
  + P(E) = frequency in favour of E / total frequency = f/n (n is large)
* **Subjective probability** – based on an educated guess/estimate from one’s experience
* Probability rules
  + 0 ≤ P(E) ≤ 1
  + P(impossible event) = 0
    - If an event has P = 0, doesn’t mean it is impossible
  + P(sure event) = 1
  + Sum of the probabilities of all outcomes in the sample space = 1
    - i.e. P(S) = 1
  + P(E) + P(!E) = 1
* **Mutually exclusive/disjoint** – 2 events cannot occur at the same time
  + P(A ∩ B) = 0
* Addition rules
  + If A & B are mutually exclusive, P(A ∪ B) = P(A) + P(B)
  + Otherwise, P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
* Counting rules
  + Combinations – pick k elements out of n elements; order does not matter
    - C(n, k) = n!/k!⋅(n – k)!
  + Permutations – pick k elements out of n elements; order does matter
    - P(n, k) = n!/(n – k)!
  + Ex. 5 men & 3 women sit in a row; probability of same gender sitting at both ends?
    - Men or women sitting at both ends – 2 cases mutually exclusive
    - n = 8!/5! ⋅ 3! = 56 – total permutation with permutations within each gender cancelled out
      * i.e. C(total #, # of men/women)
    - Men at both ends – arrange 3 men & 3 women in 6 remaining seats
      * i.e. C(6, 3) = 6!/3! ⋅ 3! = 20
    - Women at both ends – arrange 5 men & 1 woman in 6 remaining seats
      * i.e. C(6, 1) = 6!/1! ⋅ 5! = 6
    - Thus P = 20/56 + 6/56 = 13/28
* Multiplication rules
  + **Conditional probability** – P(B|A), i.e. probability of B given A
    - P(B | A) = P(A ∩ B)/P(A)
  + **Independent events** – A occurring does not affect the probability of B occurring
  + A & B are independent if & only if P(A ∩ B) = P(A) ⋅ P(B)
  + If A & B are dependent, P(A ∩ B) = P(A) ⋅ P(B | A) = P(B) ⋅ P(A | B)
    - For 3 dependent events, P(A ∩ B ∩ C) = P(A) ⋅ P(B | A) ⋅ P(C|A ∩ B)
  + **Total probability rule** – if B depends on A and Ai are mutually exclusive then
    - P(B) = ∑ P(Ai)P(B | Ai)
    - i.e. P(A1)P(B | A1) + P(A2)P(B | A2) + …
    - i.e. sum of the probabilities of B for every Ai occurring
* **Bayes’ Theorem**
  + Let A1 … Ak be mutually exclusive & exhaustive events with prior probabilities P(A1) … P(Ak)
  + If event D occurs, then P(Ai|D) = P(D ∩ Ai)/P(D) → apply total probability rule on denom.
    - i.e. P(Ax | D) = P(Ax) ⋅ P(D | Ax) / ∑ [P(Ai) ⋅ P(D | Ai)]
    - i.e. given chances of D occurring for each Ai, if D occurred, what is the probability it was Ax that occurred?
* “At least” probability
  + e.g. P(at least one) = 1 – P(none)